

# Hitchhiker's Guide to the Early Universe



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Oak Ridge

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# DON'T PANIC

# *Contents*

Evidence for a Big Bang

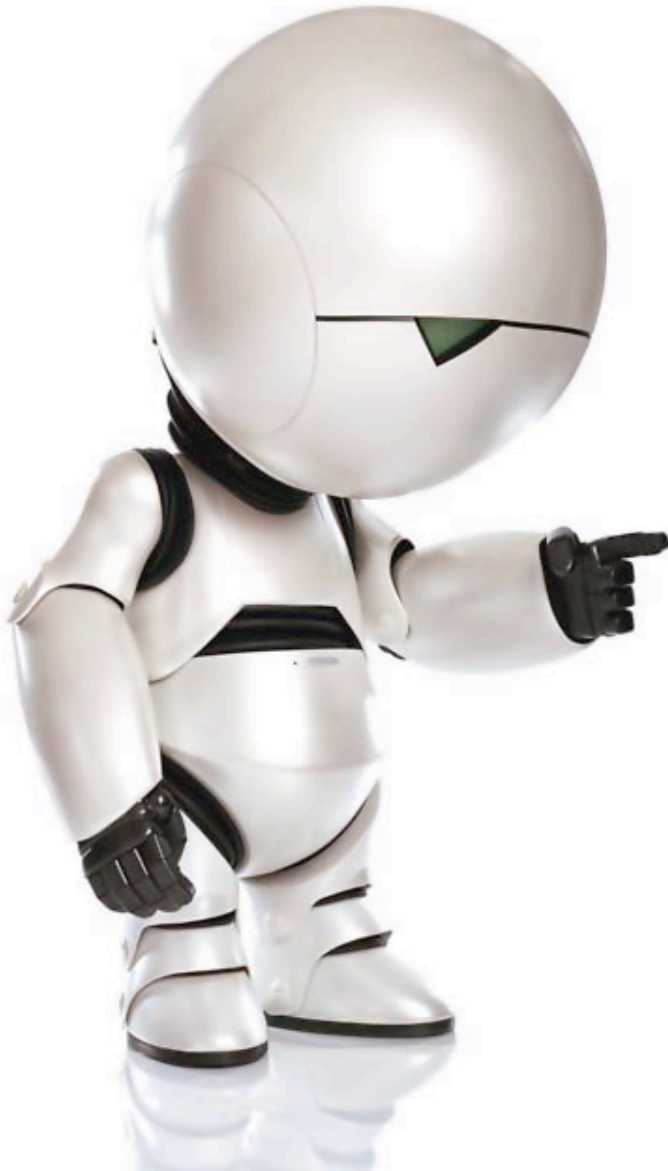
Experimental    Theoretical    Basic

Framework    A Smidgen of GR

Vacuum Energy and Inflation

Thermodynamics    Decoupling and

Relics    The QGP Transition



**DON'T PANIC**

If I can understand it,  
so can you!

# The original Hubble Diagram

“A Relation Between Distance and Radial Velocity Among Extra-Galactic Nebulae”  
E. Hubble  
(1929)

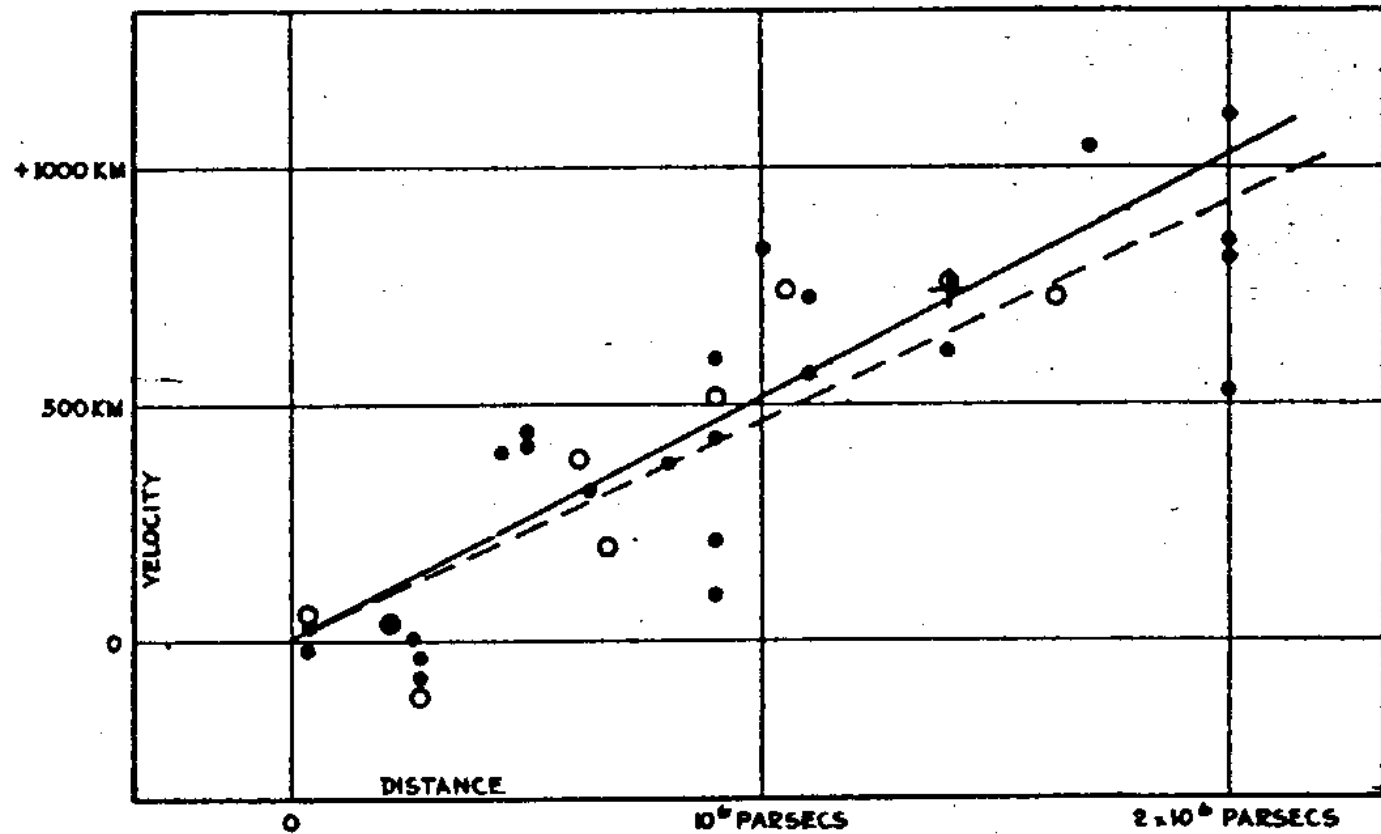


FIGURE 1



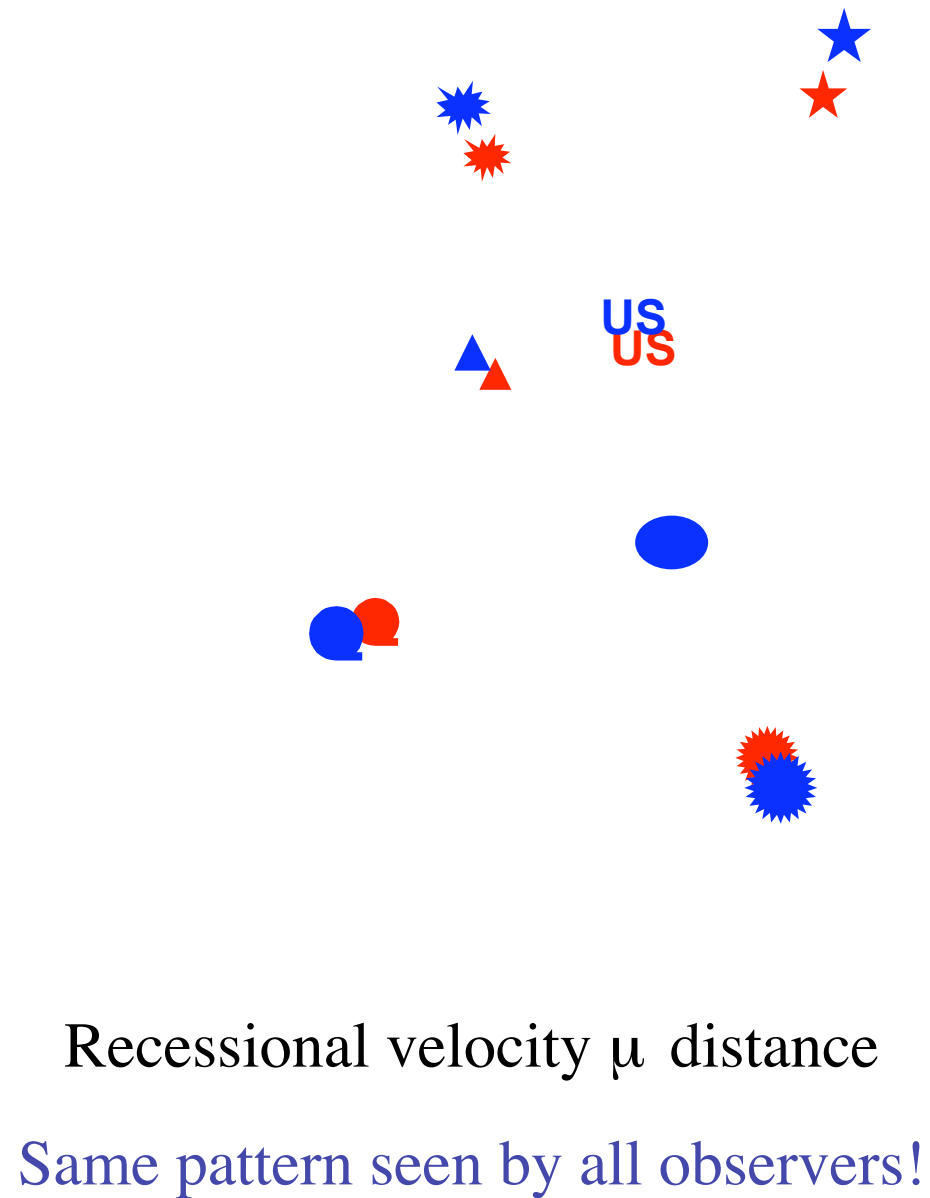
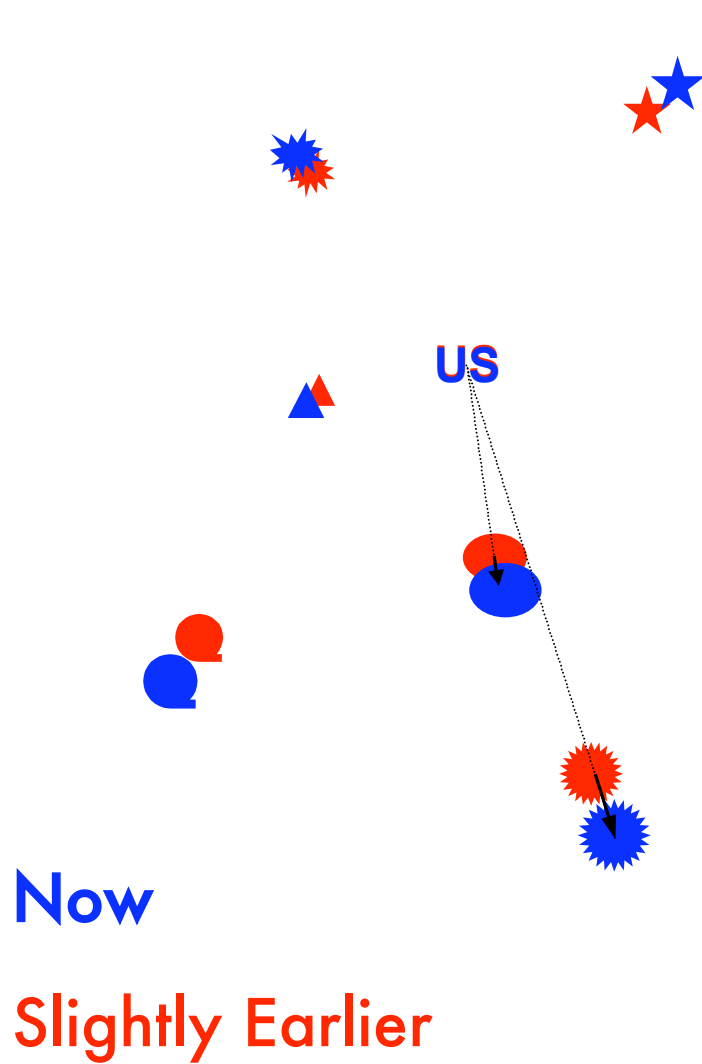
**Edwin Hubble**  
American  
Galaxies outside  
Milky Way

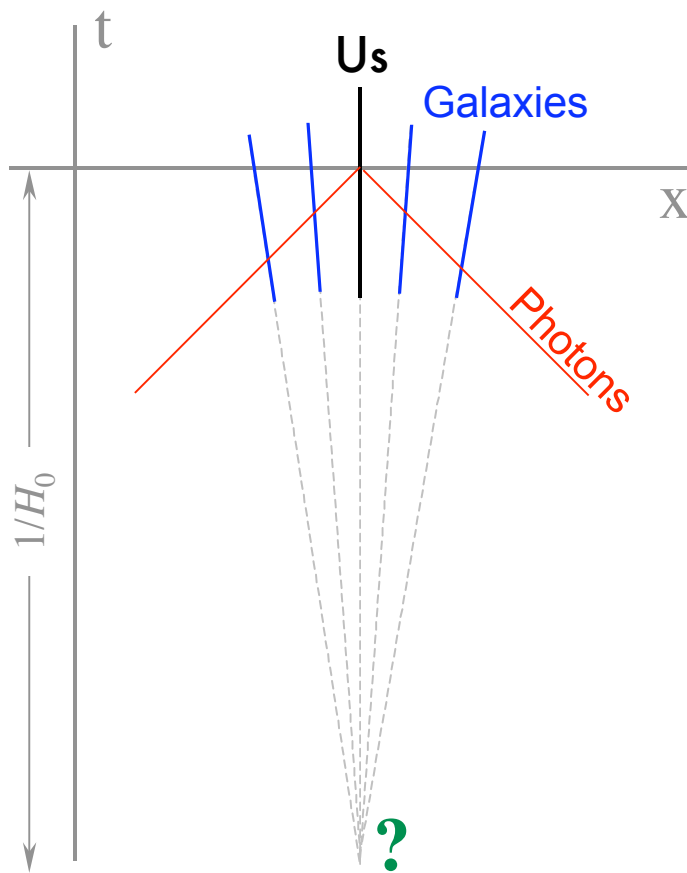


**Henrietta Leavitt**  
American  
Distances via  
variable stars

As seen from our position:

As seen from another position:





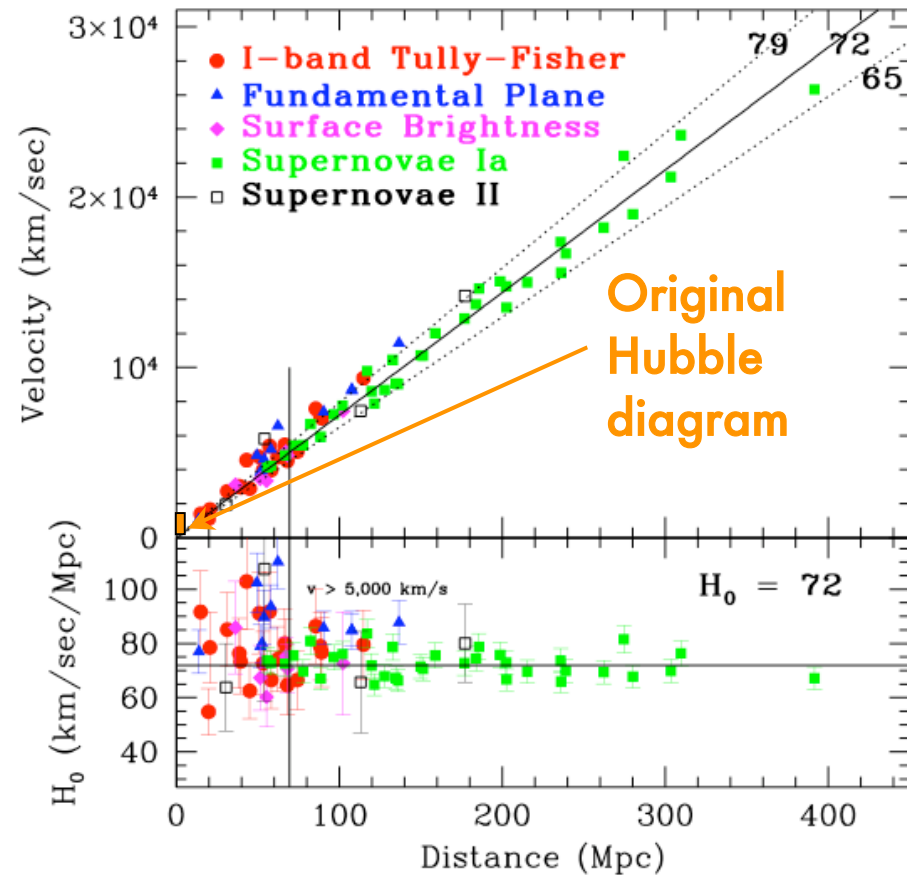
$$v_{\text{Recession}} = H_0 d$$

$1/H_0 \sim 10^{10}$  year  $\sim$  Age of the Universe?

Freedman, et al.  
Astrophys. J. **553**,  
47 (2001)

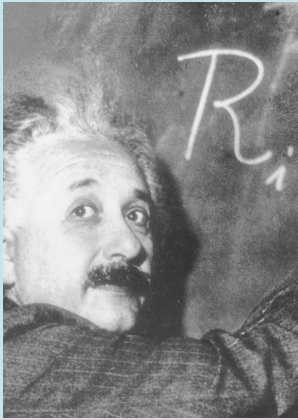
**W. Freedman**  
American

Modern Hubble  
constant (2001)



1929:  $H_0 \sim 500$  km/sec/Mpc

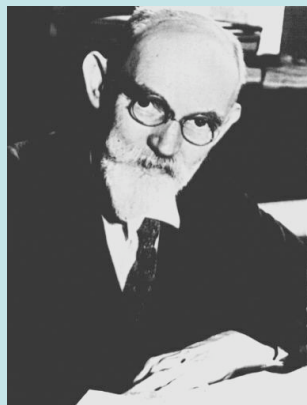
2001:  $H_0 = 72 \pm 7$  km/sec/Mpc



**Albert Einstein**

German

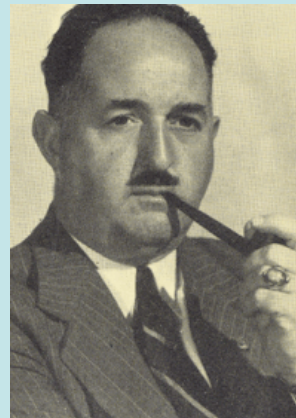
General Theory of  
Relativity (1915);  
Static, closed  
universe (1917)



**W. de Sitter**

Dutch

Vacuum-energy-  
filled universes  
“de Sitter space”  
(1917)



**H.P. Robertson**

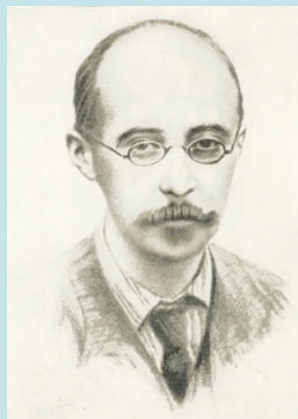
American



**A.G. Walker**

British

Formalized most general form of isotropic  
and homogeneous universe in GR  
“Robertson-Walker metric” (1935-6)



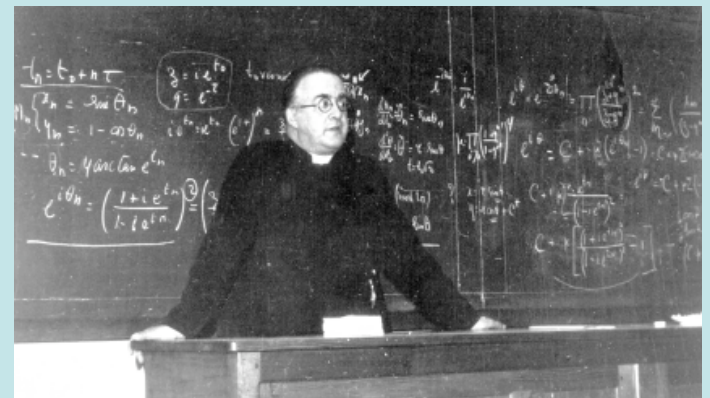
**A. Friedmann**

Russian

Evolution of homogeneous, non-  
static (expanding) universes  
“Friedmann models” (1922, 1927)

**G. LeMaitre**

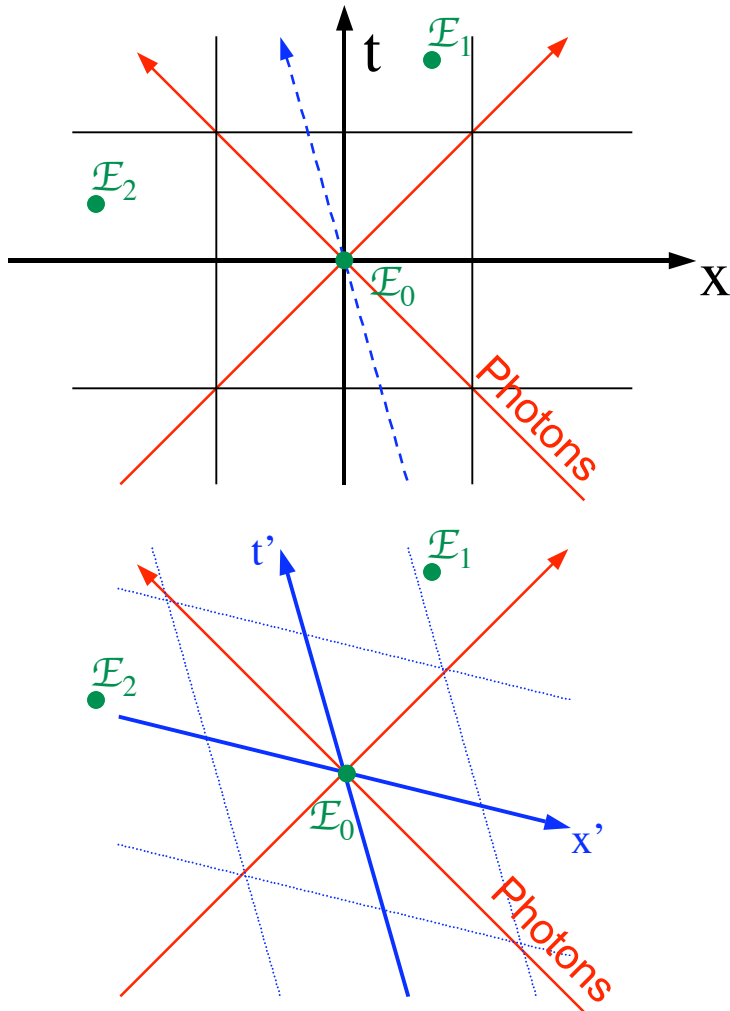
Belgian





**H. Minkowski** German

"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality" (1907)



Global  
Reference  
Frames



Convenient  
Coordinate  
Systems

$$dx^0 \equiv dt \quad dx^1 \equiv dx \quad dx^2 \equiv dy \quad dx^3 \equiv dz$$

$$d\tau^2 = -ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Metric  
Tensor

Complete coordinate freedom!  
All physics is in  $g_{\mu\nu}(x^0, x^1, x^2, x^3)$

**Homogeneity** = same at all points in space at some  $t = \text{constant}$  as seen by observers following  $dx = dy = dz = 0$

**Isotropy** = same in all spatial directions as seen from any point

(Convenient: want  $dt = d\tau$  i.e., proper/subjective time, when  $dx = dy = dz = 0$ )

## Robertson-Walker Metric:

$$d\tau^2 = \square ds^2 = dt^2 - [a(t)]^2 d\Sigma^2$$

Flat space :  $d\Sigma^2 = dx^2 + dy^2 + dz^2$

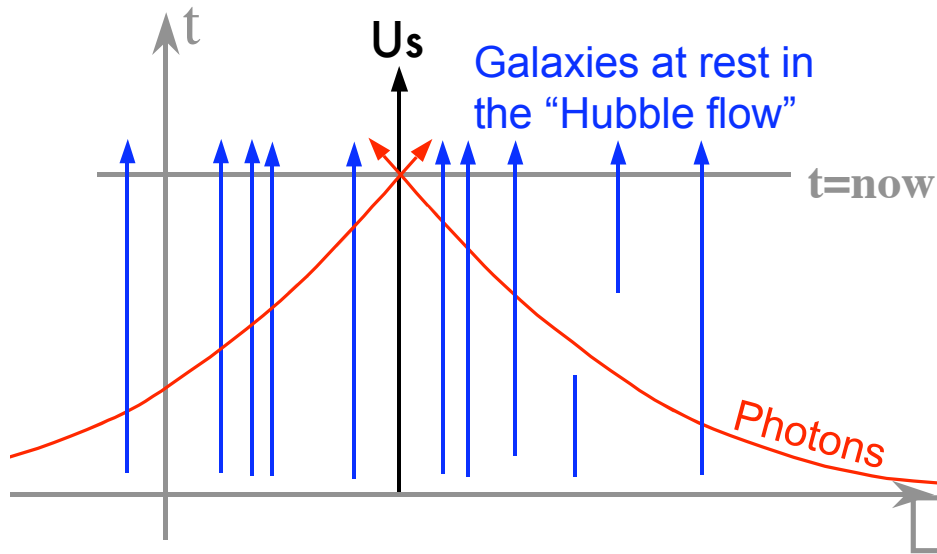
(curved space switch to  $r, \theta, \phi$ )

$a(t)$  dimensionless; choose  $a(\text{now}) = 1$

$\Sigma$  has units of length

$a(t)\Sigma = \text{physical separation at time } t$

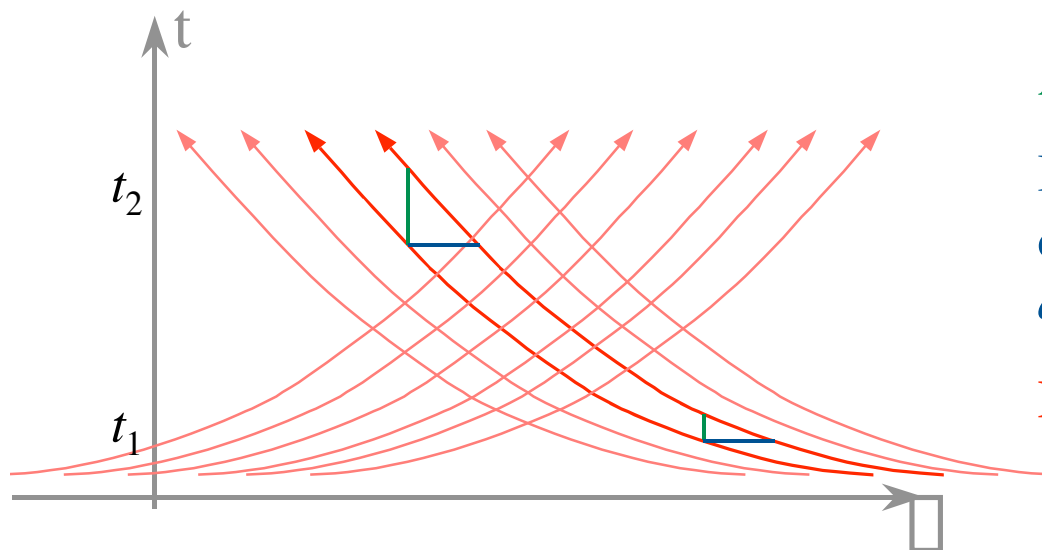
$$g_{\mu\nu}(t, x, y, z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & [a(t)]^2 & 0 & 0 \\ 0 & 0 & [a(t)]^2 & 0 \\ 0 & 0 & 0 & [a(t)]^2 \end{bmatrix}$$



Paths of  $\square = \text{constant}$  are "natural"  
 free-fall trajectories for masses  
 "at rest in the Hubble flow"

$$H(t) = \frac{\text{velocity}}{\text{distance}} = \frac{\frac{d}{dt}[a(t)\square\square]}{a(t)\square\square} = \frac{\dot{a}(t)}{a(t)}$$

Photons follow  $d\square = 0 \Rightarrow \frac{dt}{d\square} = \pm a(t)$



A photon's period grows  $\propto a(t)$

Its coordinate wavelength  $\square\square$  is  
 constant; its physical wavelength  
 $a(t)\square\square$  grows  $\propto a(t)$

**Red shift!**

$$\square(t_2)/\square(t_1) = a(t_2)/a(t_1) \equiv 1+z$$

# $a(t)$ is a Friedmann-Robertson-Walker (FRW) cosmology

Three basic solutions for  $a(t)$ :

## 1. Relativistic gas, “radiation dominated”

$$P/\rho = 1/3 \quad \rho \propto a^{-4} \quad a(t) \propto t^{1/2}$$

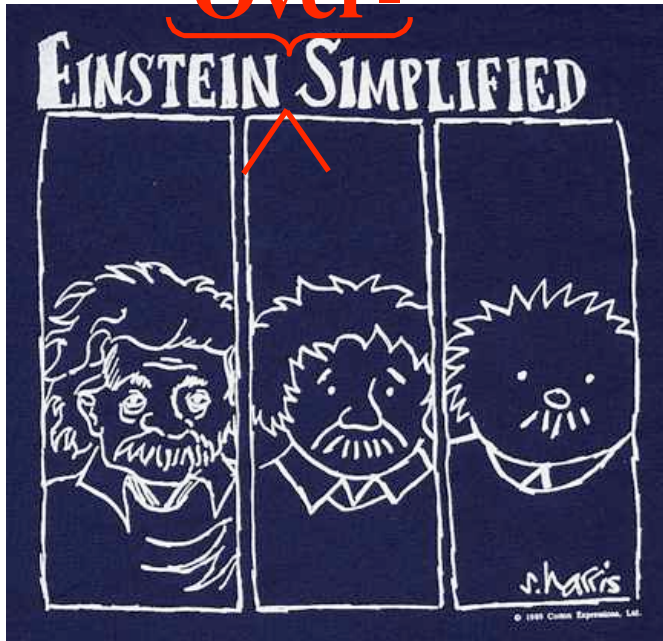
## 2. Non-relativistic gas, “matter dominated”

$$P/\rho = 0 \quad \rho \propto a^{-3} \quad a(t) \propto t^{2/3}$$

## 3. “Cosmological-constant-dominated” or “vacuum-energy-dominated”

$$P/\rho = -1 \quad \rho \propto \text{constant} \quad a(t) \propto e^{Ht} \quad \text{“de Sitter space”}$$

Over-



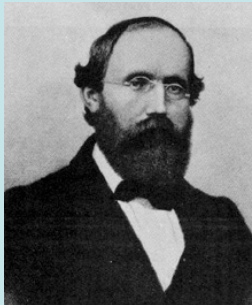
$$\begin{array}{|c|} \hline \text{Something} \\ \hline \text{about space - time} \\ \hline \text{curvature} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{Something} \\ \hline \text{about} \\ \hline \text{mass - energy} \\ \hline \end{array}$$

Metric Tensor  $g_{\mu\nu}$  Stress-Energy Tensor  $T_{\mu\nu}$

Riemann Tensor  $R^{\mu}_{\nu\alpha\beta}$

Ricci Tensor  $R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu}$

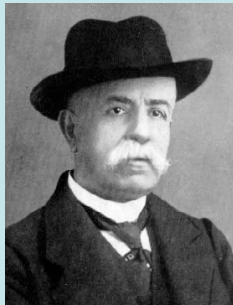
Ricci Scalar  $R = R^{\mu}_{\mu}$



**B. Riemann**

German

Formalized non-Euclidean geometry (1854)



**G. Ricci-Curbastro**

Italian

Tensor calculus (1888)

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{For continuity}} \underbrace{\quad}_{\text{Unknown}} g_{\mu\nu} = \underbrace{8\pi G_{\text{Newton}}}_{\text{To match Newton}} T_{\mu\nu}$$

$\equiv G_{\mu\nu}$  Einstein Tensor

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad \text{Einstein Field Equation(s)}$$

$$T_{00} = \rho \quad \text{Energy density (in local rest frame)}$$

## Friedmann Equation 1 ( $\Lambda=0$ version)

$$\left( \frac{\dot{a}}{a} \right)^2 = H^2 = \frac{8\pi}{3} G \rho - \frac{1}{r_0^2 a^2}$$

Curvature

$$\rho_{\text{Critical}} \equiv \frac{3H^2}{8\pi G} \quad \frac{\rho}{\rho_{\text{Cr}}} \equiv \Omega \quad \Omega = 1 \quad \text{Flat}$$

Q: How does  $\Omega$  change during expansion?

Isotropic fluid  
in local  
rest frame

$$T_{\alpha\beta} = \begin{bmatrix} \rho(t) & 0 & 0 & 0 \\ 0 & P(t) & 0 & 0 \\ 0 & 0 & P(t) & 0 \\ 0 & 0 & 0 & P(t) \end{bmatrix}$$

**Basic Thermodynamics**

$$dE = TdS + PdV$$



Sudden expansion, fluid fills empty  
space without loss of energy.

$$dE = 0 \quad PdV > 0 \quad \text{therefore} \quad dS > 0$$



Gradual expansion (equilibrium maintained),  
fluid loses energy through PdV work.

$$dE = -PdV \quad \text{therefore} \quad dS = 0 \quad \text{Isentropic}$$

## Friedmann Equation 2 (Isen/Iso-tropic fluid, $\Lambda=0$ )

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \text{Ac/De-celeration of the Universe's expansion}$$

If  $\rho(t) \geq 0$  and  $P(t) \geq 0$  then  
 $\dot{a}(t) \geq 0$  and  $\ddot{a}(t) \leq 0$ , and then  
 $a(t) = 0$  for some  $t$

**Necessity of a Big Bang!**

However, this cannot describe a static,  
non-empty FRW Universe.

## Re-introduce “cosmological constant” $\Lambda$

[illegible]

**Generalize**  $\rho(t) = \rho_{\text{Matter}}(t) + \rho/8$   $P(t) = P_{\text{Matter}}(t) - \rho/8$

Cosmological constant acts like  
constant energy density, constant  
negative pressure, with EOS  $P/\rho = -1$

## Friedmann 1 and 2:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi}{3} G \rho \frac{1}{r_0^2 a^2} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P)$$

With freedom to choose  $r_0^2$  and  $\rho$ , we can arrange to have  $\dot{a}'=\ddot{a}''=0$  universe with finite matter density

Einstein 1917 “Einstein Closed, Static Universe”

$\Lambda$  disregarded after Hubble expansion discovered

- but -

“vacuum energy” acts just like  $\Lambda$

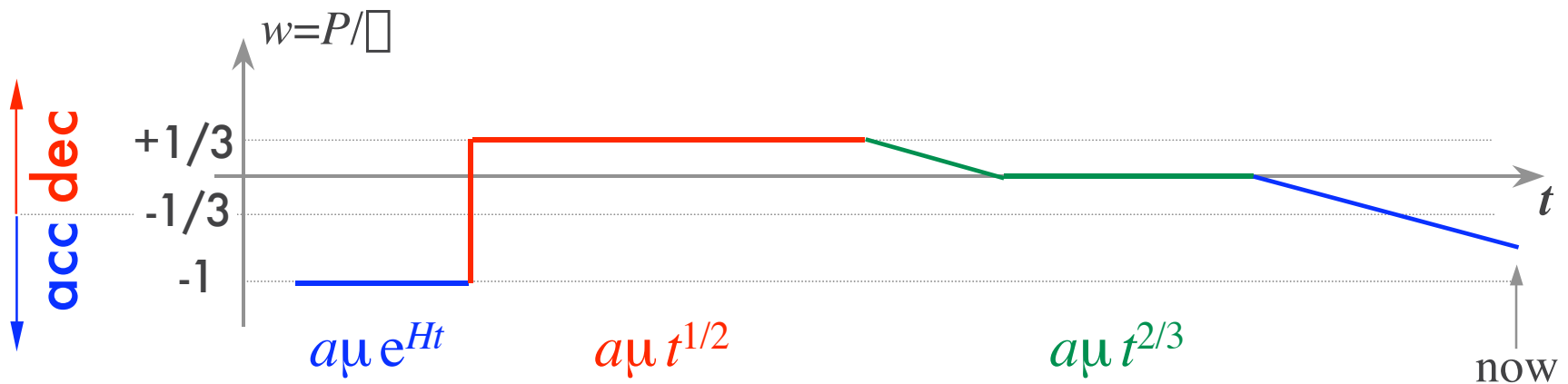
# The New Standard Cosmology in Four Easy Steps

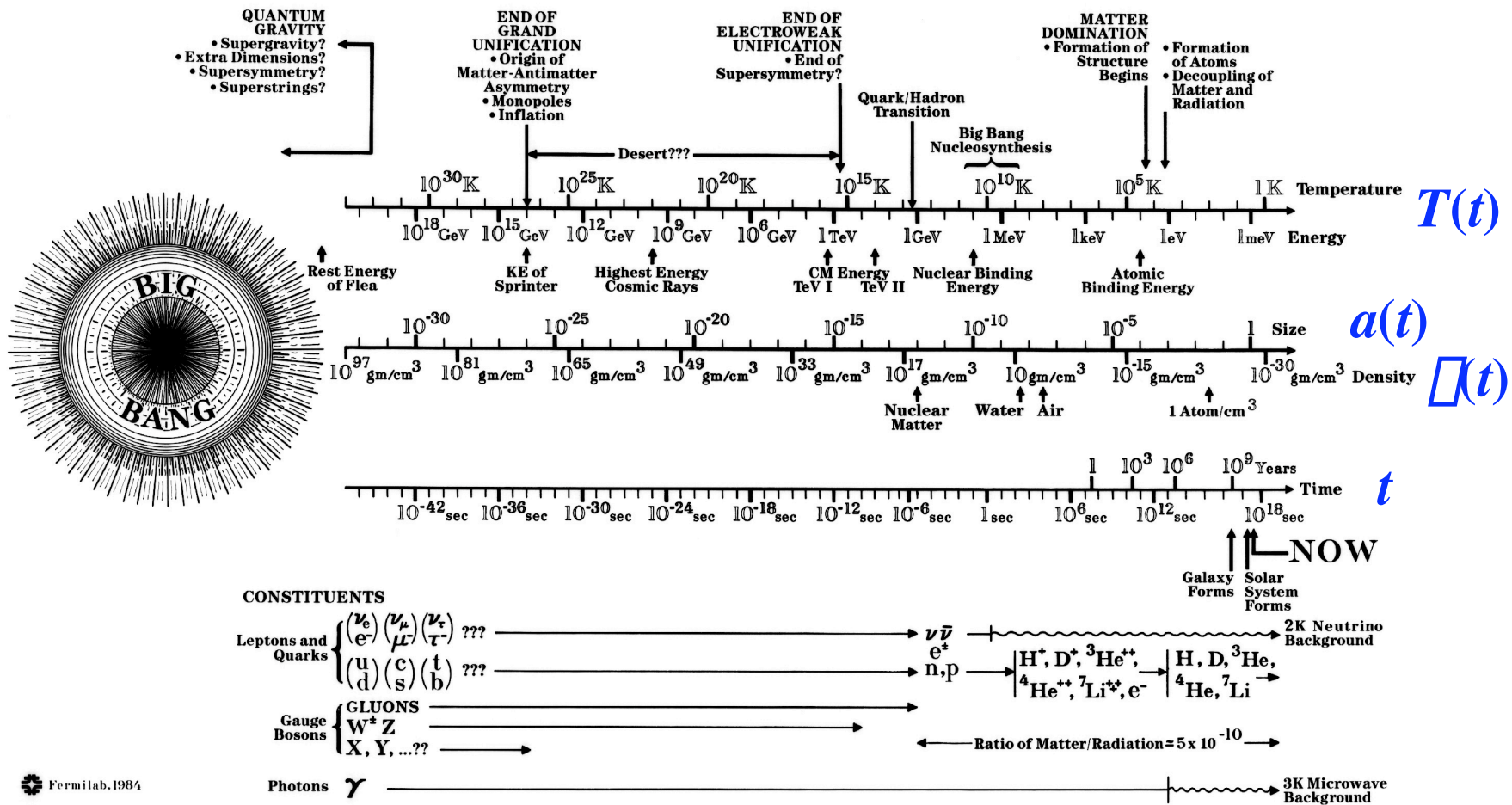
Inflation, dominated by “inflaton field” vacuum energy

Radiation-dominated thermal equilibrium

Matter-dominated, non-uniformities grow (structure)

Start of acceleration in  $a(t)$ , return to domination by cosmological constant and/or vacuum energy.





How do we relate  $T$  to  $a, \rho$  i.e. thermodynamics

Golden Rule 1: Entropy per co-moving volume is conserved

Golden Rule 2: All chemical potentials are negligible

Golden Rule 3: All entropy is in relativistic species

Expansion covers  
many decades in  $T$ ,  
so typically either  
 $T \gg m$  (relativistic) or  
 $T \ll m$  (frozen out)

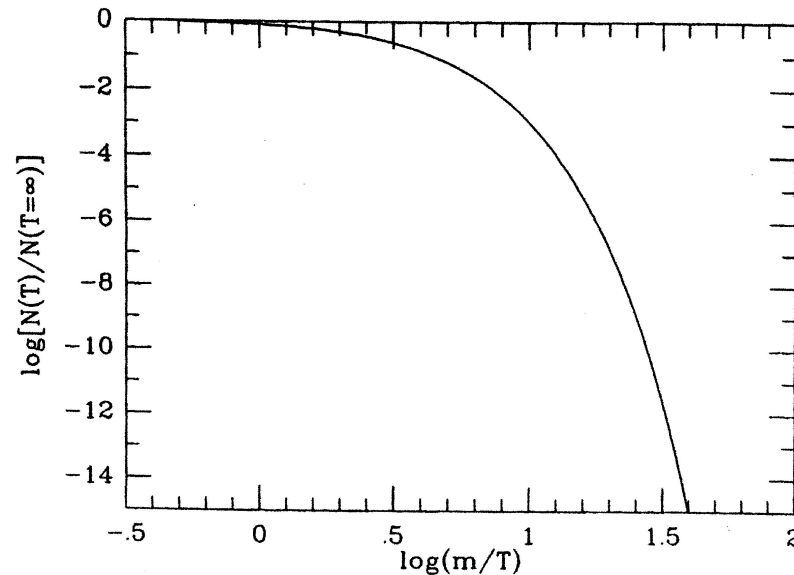


Fig. 3.6: The equilibrium abundance of a species in a comoving volume element,  $N = n/s$ . Since both  $n_\gamma$  and  $s$  vary as  $T^3$ ,  $N$  is also proportional to  $n/n_\gamma$ .

Kolb & Turner, *The Early Universe*,  
Westview 1990

Entropy  $S$  in co-moving volume  $(\bar{a}\bar{a})^3$  preserved; entropy density  $s = \frac{S}{V} = \frac{S}{(\bar{a}\bar{a})^3 a^3}$

For relativistic gas  $s = \frac{2\pi^2}{45} g_{\text{eff}} T^3$   $g_{\text{eff}} \equiv \sum_{\text{Bosons } i} g_i + \frac{7}{8} \sum_{\text{Fermions } j} g_j$  degrees of freedom

$$\frac{S}{(\bar{a}\bar{a})^3} \frac{1}{a^3} = \frac{2\pi^2}{45} g_{\text{eff}} T^3$$

Golden Rule 4:

$$T \propto (g_{\text{eff}})^{1/3} \frac{1}{a}$$

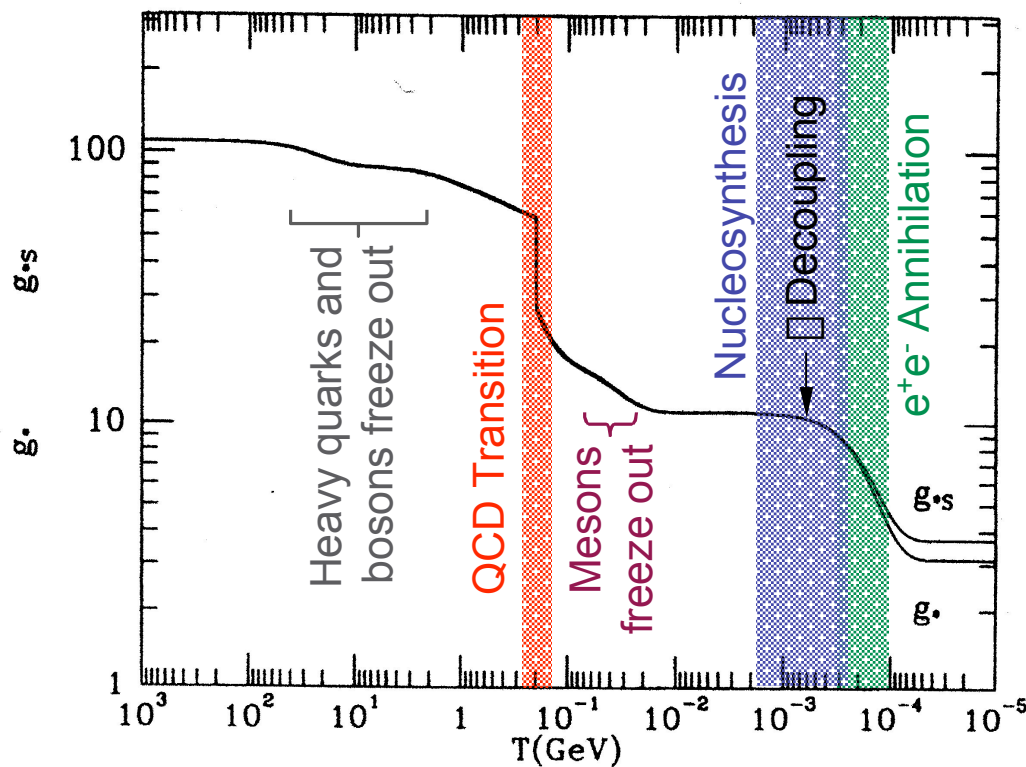


Fig. 3.5: The evolution of  $g_{\text{eff}}(T)$  as a function of temperature in the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  theory.

Kolb & Turner

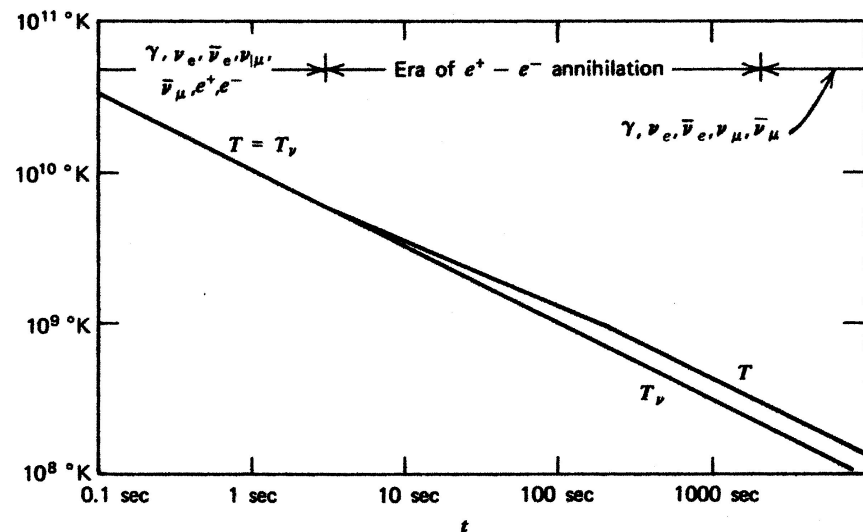
# Golden Rule 5: Equilibrium is boring!

Would you like to live in thermal equilibrium at 2.75°K?

That which survives:

- (1) **Relics**  $T > m$  but  $\dot{\rho} > H$   
(CMB photons,  
neutrinos, gravitons, **dark matter?** free quarks,  
magnetic monopoles...)
- (2) **Remnants**  $T < m$  but  $\dot{\rho} \neq 0$   
(baryons  $\dot{\rho}_B \sim 10^{10}$ ,  
electrons, **dark matter?**)

Example of  $e^+e^-$  annihilation transferring entropy to photons, after neutrinos have already decoupled (relics).



**Figure 15.5** Thermal history of the early universe. Here  $T$  is the temperature of the  $\gamma - e^+ - e^-$  plasma, and  $T_\gamma$  is the temperature of the decoupled  $\nu_e, \bar{\nu}_e, \nu_\mu$ , and  $\bar{\nu}_\mu$ .

Weinberg, *Gravitation and Cosmology*,  
Wiley 1972

## The QCD quark-hadron transition is typically ignored by cosmologists as uninteresting

**Weinberg (1972):** Considers Hagedorn-style limiting-temperature model, leads to  $a(t) \propto t^{2/3} |\ln t|^{1/2}$ ; but concludes “...the present contents...depends only on the entropy per baryon.... In order to learn something about the behavior of the universe before the temperature dropped below  $10^{12}$ °K we need to look for fossils [relics]....”

**Kolb & Turner (1990):** “While we will not discuss the quark/hadron transition, the details and the nature (1st order, 2nd order, etc.) of this transition are of some cosmological interest, as local inhomogeneities in the baryon number density could possibly affect...primordial nucleosynthesis...”

# References

Freedman & Turner, “Measuring and understanding the universe”, Rev Mod Phys 75, 1433 (2003)

Kolb & Turner, *The Early Universe*, Westview (1990)

Dodelson, *Modern Cosmology*, Academic Press (2003)

Weinberg, *Gravitation and Cosmology*, Wiley (1972)

Schutz, *A First Course in General Relativity*, Cambridge (1985)

Misner, Thorne, Wheeler, *Gravitation*, W.H.Freeman (1973)

Bartelmann & Schneider, “Weak Gravitational Lensing”, Physics Reports 340, 291-472 (2001)